Written exam at the Department of Economics
Summer 2021
Economic growth
Final exam (re-exam), August 12

3-hour closed book exam. Answers only in English.

SOLUTION MANUAL

Falling ill during the exam
If you fall ill during an examination at Peter Bangsvej, you must:

- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!
You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

1 Short essay questions

Question 1.1

Based on the readings of this course, briefly discuss whether further movement toward free trade is desirable.

Solution: The papers by Frankel and Romer (1999) and Feyrer (2009) both find positive associations between trade and GDP per capita, and in the latter case, the association is identified as being causal. Moreover, Bloom et al (2016) find that import competition from China leads to increased innovation and reallocation from low-productivity firms to high-productivity firms. All these findings point to a positive effect of free trade. However, Bloom et al (2016) and Autor (2015) also point to distributional effects: there are winners and losers from trade. International trade might therefore increase inequality, at least in the short and medium run, which leads to a negative effect on welfare which might larger than the positive effect of trade on average incomes. Increased inequality might also lead to political instability and inefficient redistribution, which will be detrimental to growth (Alesina and Rodrik, 1994). One might add that trade-induced inequality might be alleviated, at least partly, by redirecting some of the overall gains from trade towards those who are negatively affected. A reasonable conjecture is, therefore, that trade on the balance is positive for welfare if appropriate institutions are in place.

Question 1.2

What is an O-ring production function? Explain why such a production function may entail a correlation between the human capital of a firms' workers and the complexity of its products.

Solution: An O-ring production function is a production function in which a number of tasks need to be completed in order to produce one unit of output. If any of these tasks fails, the produced output will be worthless. Suppose that workers can either succeed or fail at their tasks, that each worker perform exactly one task, and that complexity is captured by the number of tasks needed to finish one unit of output. The cost of failing will be larger the larger the number of tasks required to finish a product, as the failure of one worker will eliminate the value created by all other workers contributing to the product. If a higher level of human capital is associated with a greater likelihood of succeeding, the (expected) marginal value of hiring a high-skill worker will be higher the greater in firms producing complex products. This effect will lead to a concentration of skilled workers in industries with complex products.

Question 1.3

Suppose new technologies gradually make more and more jobs redundant because of automation. Explain why workers might not be adversely affected by this process in the long run. Be specific about the mechanisms at work.

Solution: In the framework of Acemoglu and Restrepo, the negative displacement effect of automation can be offset by a productivity effect. Essentially, the productivity gains associated with automation reduces relative prices of the affected tasks, leading to increases in real wages. Which effect dominates is theoretically ambiguous, but capital deepening will tend to increase the productivity effect relative to the displacement effect. Students might also mention that ongoing technical change as well as the creation of new tasks might benefit workers. Additionally, as argued by Autor (2015), there might be complementarities between machines and workers in certain tasks, resulting in increased demand for labor in those tasks.

Question 1.4

Postulate: "If the aggregate production function exhibits constant returns to scale in the reproducible factor of production (asymptotically, at least) then positive growth in income per capita is ensured". Do you agree or disagree? Explain why/why not.

Solution: Disagree. The statement involves the requirement for (strictly) endogenous growth. But this is about feasibility, not whether in fact growth is positive in the long-run. For instance, in a Ramsey-Cass-Koopmans model where the aggregate production function fulfills the mentioned criteria growth is given by the consumption Euler equation, so that $\gamma = \frac{1}{\theta}(r - \rho)$ where the notation is standard. Constant returns to reproducible factor ensures r is constant, but not that $r > \rho$. Perhaps more profoundly, if we study a Diamond framework where people thus save from labor income growth can come to a halt if the share of labor asymptotes to zero - which is what it will do in a competitive setting if one imposes asymptotically constant returns to the reproducible factor - capital in this case (see the paper by Jones and Manuelli on the reading list). This is occurs because savings behavior is very different from the RCK setting, where people can be seen as featuring a perfect bequest motive and thus save from capital income. Bottom line: whether growth actually occurs depends on preferences as much as technological requirements.

Question 1.5

There is currently an active debate on the future of economic growth. One argument, originally associated with the economist Robert Gordon, is that productivity growth is likely to decline because we already have harvested the "low hanging fruits", technologically speaking. Explain how one can turn this somewhat vague notion into a testable implication, and whether the data supports Robert Gordons' hypothesis.

Solution: The statement can be translated into a more formal statement by employing the notion of an idea-production function. Suppose growth of new ideas, $\dot{A/A}$ are produced by R&D labor in the following way

$$\dot{A}(t)/A(t) = \delta(t)L_A(t)$$

where $L_A$is research labor and δ is R&D productivity, which in turn is given by

$$\delta(t) = dA(t)^{\phi-1}$$

with $\phi < 1$. Accordingly, as the stock of knowledge expands R&D productivity, δ, declines. This effect, which in growth theory usually is referred to as "the fishing out effect", suggests that the only way to maintain a constant growth of ideas is by spending progressively more resources on R&D; in the setting above, manifested in more R&D labor. If R&D labor grow exponentially growth in the stock of new ideas is exponential. Bloom et al (on the reading list) investigate whether this has been taking place over the last several decades and find that R&D productivity in fact has been declining, consistent with the predictions of the model above. Hence, unless R&D input grows sufficiently (and eventually it is probably infeasible if it requires constant growth of labor input) growth will slow down as hypothesized by Robert Gordon.

Question 1.6

Suppose a group of countries establish a "common market", which ensures a fully integrated labor market and more competition over-all. Imagine these are the only effects of the common market. Will the common market initiative necessarily increase growth in the member state countries according to an Aghion-Howitt model? Explain why/why not.

Solution: No, not necessarily. On the one hand a common market will expand the total labor force and thus allow for more innovation though the scale-effect, which is present in the Aghion-Howitt model. Accordingly, if the incentive to conduct R&D was unaffected otherwise by the common market, growth would increase. But if we see more competition the gains from innovating will decline since it lowers the monopoly profits that can be attained through innovation; there is more creative destruction. As a result, the net effect is ambiguous.

2 Skill biased technical change in the task-based model

Consider the task-based model in Autor and Acemoglu (2011). Aggregate production is a Cobb-Douglas aggregate of a continuum of distinct tasks:

$$Y = \exp\left[\int_0^1 \ln y\left(i\right) di\right]$$

Each task is produced according to

$$y\left(i\right) = A_L \alpha_L\left(i\right) l\left(i\right) + A_M \alpha_M\left(i\right) m\left(i\right) + A_H \alpha_H\left(i\right) h\left(i\right)$$

where $l(i)$, $m(i)$ and $h(i)$ denote low-skilled labor, medium-skilled labor, and high skilled labor, respectively, employed in producing task i. A_L, A_M, A_H are general skill-specific productivity levels, and α_L, α_M and α_H are task *and* skill-specific productivity levels. Let $p(i)$ denote the price of task i. Assume that $\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing such that low-skilled workers produce tasks $i < I_L$, high-skilled workers produce tasks $i > I_H$, and medium-skilled workers produce tasks $I_L \leq i \leq I_H$. Lastly, labor markets clear such that:

$$\int_0^1 l(i)\,di = L, \quad \int_0^1 m(i)\,di = M, \quad \textit{and} \quad \int_0^1 h(i)\,di = H,$$

Question 2.1

Show that the equilibrium labor allocation is as follows:

$$\begin{aligned} l(i) &= \frac{L}{I_L} \text{ for any } i < I_L \\ m(i) &= \frac{M}{I_H - I_L} \text{ for any } L_L < i < I_H \\ h(i) &= \frac{H}{1 - I_H} \text{ for any } i > I_H \end{aligned}$$

Hint: You will need to derive the wage rates for each type of worker, and use the property of the symmetric Cobb-Douglas production function that expenditures on each task are identical.

Solution: The first step is to derive wages from the first order conditions for each task $i \epsilon I$

$$\begin{array}{cc} w_L = p(i)\,A_L\alpha_L(i) & \text{for any } i \leq I_L \\ w_M = p(i)\,A_M\alpha_M(i) & \text{for any } I_L \leq i \leq I_H \\ w_H = p(i)\,A_H\alpha_H(i) & \text{for any } i \geq I_H \end{array}$$

These conditions say that marginal costs should equal marginal revenue. Now, we know that production of final output is Cobb-Douglas and symmetric across tasks, so expenditures on all tasks are equal. By implication, for any $i, i\prime < I_L$, we have that::

$$\begin{aligned} p(i)\,y(i) &= p(i')\,y(i') \\ \Leftrightarrow p(i)\,A_L\alpha_L(i)\,l(i) &= p(i')\,A_L\alpha_L(i')\,l(i') \\ \Leftrightarrow w_L l(i) &= w_L l(i') \\ \Leftrightarrow l(i) &= l(i') \end{aligned}$$

This expression shows that the same amount of labor are employed in all tasks produced by low-skill labor. We can now write $l(i)$ as the total number of low-skill workers divided by the number of tasks produced by low-skill labor (which is simply I_L):

$$l(i) = \frac{L}{I_L}.$$

The expressions for $m(i)$ and $h(i)$ can be derived in the same way.

Question 2.2

Derive the two no-arbitage conditions for labor at I_L and I_H:

$$\frac{A_M \alpha_M (I_H) M}{I_H - I_L} = \frac{A_H \alpha_H (I_H) H}{1 - I_H} \tag{1}$$

$$\frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L} \tag{2}$$

Solution: Producers of $y(I_L)$ should be indifferent between using low skill and medium skill labor, which they will be if the labor costs of the two types of labor are the same:

$$\begin{aligned} w_L l(I_L) &= w_M m(I_L) \\ &\Leftrightarrow \; p(I_L) A_L \alpha_L (I_L) \frac{L}{I_L} = p(I_L) A_M \alpha_M (I_L) \frac{M}{I_H - I_L} \\ &\Leftrightarrow \; \frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L} \end{aligned}$$

The no-arbitrage condition at I_H can be found in the same way.

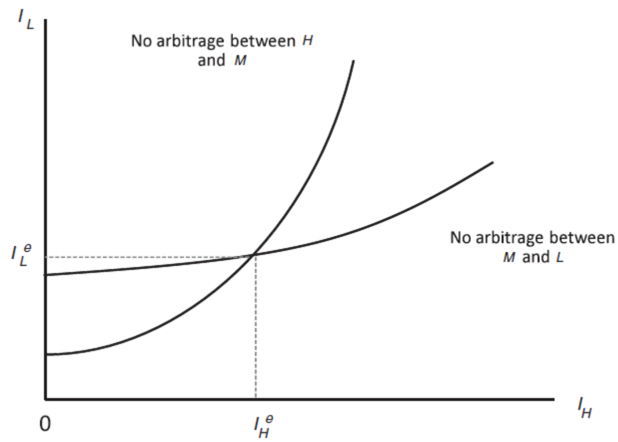
Question 2.3

Suppose that medium-skilled workers are replaced by machines in some of the tasks they produce. Explain intuitively what happens to the two cut-off values I_H and I_L. Use your answer and the no-arbitage conditions to illustrate what happens in a (I_H, I_L)-diagram.

Solution: The stock of suddenly redundant medium skill workers put downward pressure on medium skill wages. Firms respond by substituting high and low skill labor for medium skill labor in occupations where medium skill workers were relatively close to being competitive before the shock, and in which they now are competitive. This effect reduces I_L and increases I_H.

The two no-arbitrage conditions can be plotted as in Figure 1. The replacement of medium-skilled labor by machines shifts the no-arbitrage curve for M and L downward and the no-arbitrage curve for H and M leftward.

Figure 1: Equilibrium in the task based model



Question 2.4:

Based on your answer to the previous question, explain (in words) what happens to the relative wage rates $\frac{w_H}{wM}$ and $\frac{w_M}{w_L}$. Is the model's predictions for $I_L, I_H, \frac{w_H}{wM}$, and $\frac{w_M}{w_L}$ consistent with what we observe in the data? Discuss.

Solution: That medium skilled workers become employed in new occupations does not compensate for the initial wage loss, as their comparative advantages in the new occupations are smaller than in the occupations taken over by machines. By contrast, low skill workers and high skill workers increasingly become concentrated in tasks in which their comparative advantages are stronger, and the relative wage of medium skilled workers therefore declines vis-à-vis the other two groups ($\frac{w_H}{wM} \uparrow$ and $\frac{w_M}{w_L} \downarrow$). The effects on the two cut-offs are consistent with the employment polarization observed in the data (i.e., that employment in medium skill jobs has declined, while employment in low skill and medium skill jobs have increased. The wage effects are consistent with the observed wage polarization (i.e., that wages of both low skill workers and high skill workers have increased relative to medium skill workers). These effects are consistent with ICT and robots mainly replacing medium skill workers such as secretaries and production workers. Many low skill jobs, such as cleaning, are harder to automate as they are non-routine. High skill jobs are often also hard to automate, and especially ICT might even be complementary to high skill workers (e.g., researchers). The replacement of medium skill workers by machines is therefore a plausible scenario, and the model delivers reasonable predictions regarding its consequences.

3 Accidental bequests and inequality

Consider an overlapping generations economy where economic activity extends into the infinite future. The economy is closed and all markets are competitive. All individuals survive at most for two periods. During the first period people work and decide on how much to save for retirement. During this period they also have an off-spring (costlessly), which will ensure the population always is populated. There is no population growth, so each young person "spawns" one off-spring. The size of each generation is normalized to one to keep notation simple.

Surviving to retirement is not guaranteed however. Only with probability π will the individual manage to experience retirement after which they die for sure, as in a standard Diamond model. With probability $(1-\pi)$ they die prematurely. This will leave unclaimed savings, since this fraction of people do not get to consume the savings they build up during their working years. We assume that this income flow is simply passed on to their off-spring as "accidental bequest". Notice that this creates inequality since some households receive bequest while others do not. We will study the dynamics of inequality below.

Individual i maximizes expected utility

$$\ln c_{1t}^{i} + \pi \ln c_{2t+1}^{i},$$

The first period budget constraint is $c_{1t}^{i} + s_t^i = w_t^i + b_t^i \equiv I_t$ and $c_{2t+1}^{i} = (1+r_{t+1})\, s_t^i$. The notation is c for consumption (subscript 1 for consumption during youth and 2 for consumption during old age), s is savings of the young, b is an accidental bequest, and w is lifetime wage income. It can be shown that expected or average (since the is no aggregate uncertainty) savings of the young is:

$$E\left(s_t^i\right) = s_t = \frac{\pi}{1+\pi} E\left(w_t^i + b_t^i\right) = \frac{\pi}{1+\pi}\left(w_t + b_t\right). \tag{3}$$

Expected - or average - bequest (since the is no aggregate uncertainty) is given by:

$$E(b_t) = b_t = (1-\pi)\,(1+r_t)\, E\left(s_{t-1}\right) = (1-\pi)\,(1+r_t)\, s_{t-1}, \tag{4}$$

where s_{t-1} is average savings of the "parent" generation. So average bequest is the probability of premature death multiplied by the capitalized value of the savings of their "parents".

Denote by $\sigma_{b,t}^2$ as the variance of bequests at time t Assume that the expected (average) wage is $E(w_t^i) = w_t$ and that the variance of wage income at time t is σ_w^2 and constant over time. Moreover, we assume that there is *no* correlation between individual level bequests and individual level wage income.

Question 3.1

Assume the real rate of return is constant over time, $r_t = r$, and that wages grow at the rate of technological change, $A_t = (1+g)\, A_{t-1}$. Show that average bequests in efficiency units are

$$\tilde{b}_t = (1-\pi)\,\frac{(1+r_t)}{1+g}\,\frac{\pi}{1+\pi}\left(\tilde{w}_{t-1}+\tilde{b}_{t-1}\right)$$

where the generic variable $\tilde{x_t} = x_t/A_t$.

Solution: Inserting equation (3) into equation (4) yields

$$b_t = (1-\pi)\,(1+r)\,\frac{\pi}{1+\pi}\,(w_{t-1}+b_{t-1})$$

Next divide through by A_t

$$b_t = (1-\pi)\,(1+r)\,\frac{\pi}{1+\pi}\left(\frac{w_{t-1}}{A_{t-1}}\frac{A_{t-1}}{A_t}+\frac{b_{t-1}}{A_t}\frac{A_{t-1}}{A_t}\right)$$

using that $A_t = (1+g)\,A_{t-1}$ and rearranging leads to the stated result.

Question 3.2

Using the above equation and the information given in the assignment, show that bequest inequality (in efficiency units) measured by the variance of $\tilde{b}_t$ is:

$$\sigma^2_{\tilde{b},t} = \left[\frac{(1-\pi)\,\pi}{1+\pi}\frac{1+r}{1+g}\right]^2\sigma^2_{\tilde{w}} + \left[\frac{(1-\pi)\,\pi}{1+\pi}\frac{1+r}{1+g}\right]^2\sigma^2_{\tilde{b},t-1} \tag{5}$$

(Hint: suppose X, Z, Y are stochastic variables, where $X = aY + bZ$ and a, b are constants. Then the following holds: $VAR(X) = a^2VAR(Y) + b^2VAR(Z) + 2COV(aY, bZ)$, where the last term is the covariance between the two right hand side variables.)

Solution: Using the formula - and recalling that w and b are assumed to be *uncorrelated* - we get the expression given in the question.

Question 3.3

Assume $r > g$. Draw the transition diagram for the model, using the dynamic equation you just derived (Equation 5). Under what parameter restriction does a unique stable steady state exist? Show that the steady state variance of bequest is

$$\sigma^2_{\tilde{b}} = \frac{\left[\frac{(1-\pi)\pi}{1+\pi}\frac{1+r}{1+g}\right]^2}{1-\left[\frac{(1-\pi)\pi}{1+\pi}\frac{1+r}{1+g}\right]^2}\sigma^2_{\tilde{w}}$$

Solution: The dynamic equation is drawn in a $\left(\sigma^2_{\tilde{b},t-1}, \sigma^2_{\tilde{b},t}\right)$ diagram along with the steady state condition $\sigma^2_{\tilde{b},t-1} = \sigma^2_{\tilde{b},t}$. The dynamic equation is linear with a positive intersection with the vertical axis at $\left[\frac{(1-\pi)\pi}{1+\pi}\frac{1+r}{1+g}\right]^2\sigma^2_{\tilde{w}}$. The slope of the dynamic equation must be less than unity to ensure that a unique, steady steady state exists (i.e., it must intersect the 45-degree line given by $\sigma^2_{b,t-1} = \sigma^2_{b,t}$). In other words, it must be the case that:

$$\frac{(1-\pi)\,\pi}{1+\pi}\frac{1+r}{1+g} < 1$$

or:

$$\frac{(1-\pi)\,\pi}{1+\pi} < \frac{1+g}{1+r}.$$

This requires π (the survival probability to retirement) to be sufficiently high. The steady state for $\sigma^2_{\bar{b}}$ can be derived by plugging $\sigma^2_{b,t-1} = \sigma^2_{b,t} = \sigma^2_{\bar{b}}$ into Equation 5 and solving for $\sigma^2_{\bar{b}}$.

Question 3.4

In the last question we assumed $r > g$. Is this assumption an empirically reasonable, as a general regularity? Explain why you say yes/no.

Solution: Yes, in the readings the article by Jorda et al. shows that average capital returns - in practice a weighted sum of returns on equity, housing as well as bonds - has exceeded GDP growth on average over the last 100 years (give or take). One may note that this is true among a set of "large" economies, such as US, Germany and France. There are exceptions though; in particular during the first and second world war where $r < g$ appears to hold. But in general $r > g$.

Question 3.5

Thomas Piketty has argued that $r > g$ leads to greater inequality. What is the impact from an increase in $r - g$ on bequest inequality, according to the present model? Provide an economic interpretation of the result.

Solution: It is plain to see that if $\frac{1+r}{1+g}$ rises so does $\sigma^2_{\bar{b}}$.The economic intuition is that when the interest increases this works to increase the variance in bequest since the accumulated gain from receiving bequest increases. If the interest rate increases relative to the growth rate it increases the variation in bequest relative to the mean. One can say that $r > g$ enhances existing sources of inequality (differential bequest and wage income). A r > g does not in itself lead to rising inequality. But it enhances inequality, and if the "gap" increases inequality goes up ceteris paribus. Piketty usually notes that inequality has exhibited a U shaped path over the 20th century, with a trough mid century. Piketty ascribes this to the two world wars, which also are periods where r-g are "low" - negative in fact.